NASA TECH BRIEF

Lewis Research Center

Indefinite Integrals of Products of Some Exponential and Trigonometric Functions

Two integrals have been developed which are needed to solve certain problems of mathematical physics related to the Bessel and Euler equations. These integrals are:

$$\int x^p \cos(b \ln x)\, dx = \frac{x^{p+1}}{(p+1)^2+b^2} \left[b \sin(b \ln x) + (p+1)\cos(b \ln x) \right] + c;$$

$$\int x^p \sin(b \ln x)\, dx = \frac{x^{p+1}}{(p+1)^2+b^2} \left[(p+1)\sin(b \ln x) - b\cos(b \ln x) \right] + c$$

where x is the variable, b is an arbitrary constant, and p is unrestricted. Tables of integrals contain these integrals for p=0 and p=−1, but not for an unrestricted value of the exponent.

To obtain the above integrals, consider

$$\int x^p e^{ib \ln x} dx = \int x^{ib+p} dx = \frac{x^{ib+p+1}}{(p+1)+ib} + c$$

where $i=\sqrt{-1}$, e is the base of the natural logarithm, and c is the integration constant.

Rationalizing the denominator and expressing the exponential function in terms of the circular functions results in:

$$\int x^p e^{ib \ln x} dx = \frac{\left[(p+1)-ib\right] x^{p+1}}{(p+1)^2+b^2} \left[\cos(b \ln x) + i \sin(b \ln x) \right] + c$$

Splitting into the real and the imaginary parts yields the two integrals.

Notes:

1. Integrals of this type have known application in problems using cylindrical coordinates.
2. Documentation may be obtained from:
   Technology Utilization Officer
   Lewis Research Center
   21000 Brookpark Road
   Cleveland, Ohio 44135
   Reference: B72-10225

Patent status:

No patent action is contemplated by NASA.

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Category 09